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# ANNUAL STATUS REPORT FOR NASA GRANT NAG 5-2016

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A reported  $e^\pm$  annihilation line from the Crab pulsar [See Fig. 1] has three remarkable properties. It is

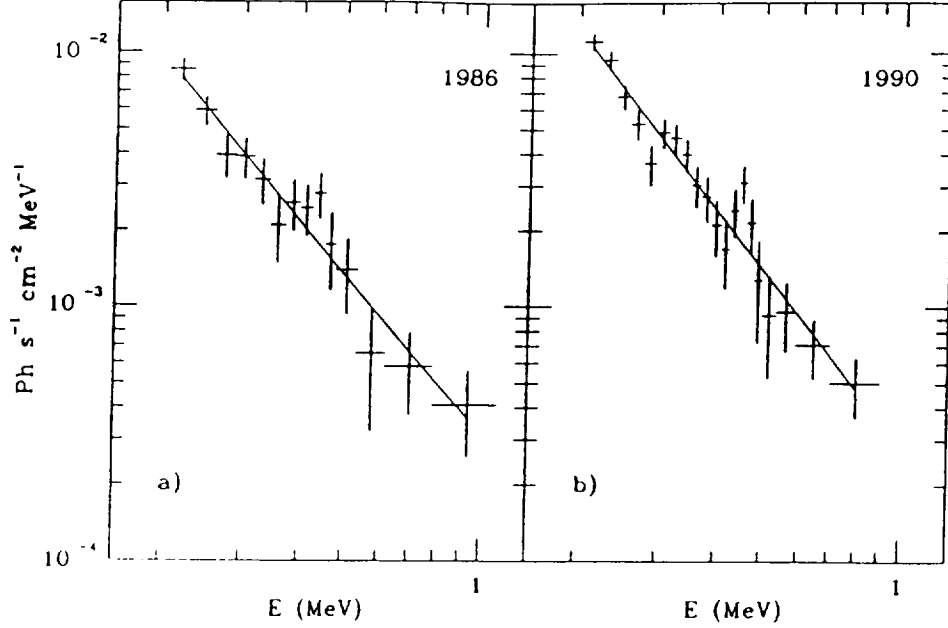
1. very strong ( $\sim 10^{40} e^\pm$  annihilations/sec.)
2. red shifted (by about 70 KeV relative to  $mc^2 = 511$  KeV)
3. very narrow (width  $\sim 10$  KeV)

A plausible model which gives all three properties has proven difficult to construct but there appears to be one. We [T. Zhu and M.R.] found the following:

- a) Polar cap accelerator models could give an abundant outflowing pair flux of about  $10^{38} \text{s}^{-1}$ , but not enough to explain property 1.
- b) Most of the  $e^\pm$  annihilation did not occur near the star in such models so that the redshift, suggestive in its magnitude of a gravitational redshift from annihilation near the stellar surface, did not have an explanation.

Therefore, we concentrated on  $e^\pm$  pair production from outer-magnetosphere accelerator models.

- c) Crossed beam geometry in such models does give abundant  $\gamma + \gamma \rightarrow e^- + e^+$ , which might be capable (barely) of maintaining  $\dot{N}_\pm \sim 10^{40} \text{s}^{-1}$  flowing down toward the stellar polar cap along the open field line bundle which joins the distant accelerator and the star (and a similar flux injected into the neutron star's wind).
- d) However, the density of that flux near the polar cap,  $n_\pm \sim 10^{20} \text{cm}^{-3}$ , would be so great that the column of annihilating  $e^+ + e^- \rightarrow \gamma + \gamma$  would not be optically thin to the escaping  $\gamma$ -rays. This would cause unacceptably large  $\gamma$ -ray energy loss (more than the "observed" 70 KeV) and unacceptable line broadening.
- e) In such models with all of the  $\dot{N}_\pm$  constrained to flow to the star along the relatively small ( $10^{10} \text{cm}^2$ ) open field line bundle near the stellar surface, pair annihilation occurs before the surface is reached. If this annihilation is before passage through a shock the large inflow speed gives annihilation line energy shifts which vary greatly through the observed pulse (and thus would give huge line broadening). If the annihilation takes place after passage through a standing shock above the surface, the post-shocked  $e^\pm$  plasma is much too hot to give property 3.



**Fig. 1.** The net spectra of the Crab photons after subtraction of the off-pulse signal, for the two FIGARO II flights of (a) 1986 July 11 and (b) 1990 July 9. From E. Massaro et al., *Ap.J.*, **376**, L11 (1991).

For all of these reasons we reconsidered the entire problem raised by properties 1–3. A new model emerged which at this time is the only one we know of which does not appear to be incompatible with the above properties 1) – 3).

The total X-ray and  $\gamma$ -ray emission from the Crab pulsar is about  $10^{-3}$  of that neutron star's spin-down power ( $-I\Omega\dot{\Omega}$ ). The scaling laws for the power from an outer-magnetosphere accelerator as a function of the fraction of the open field line bundle spanned by the accelerator then gives that fraction as  $f \sim 10^{-1}$ . The current flow through the accelerator ( $\dot{N}_a$ ) is then that same fraction  $f$  of the total Goldreich-Julian current flow. For the Crab pulsar this flow is

$$\dot{N}_a \sim f\pi BR^3\Omega^2 e^{-1} c^{-2} f \cdot 10^{34} \text{s}^{-1} \sim 10^{33} \text{s}^{-1}. \quad (1)$$

Within the accelerator this  $\dot{N}_a$  flow is accelerated by an electric field along  $\vec{B}$  of magnitude  $|\vec{E}| \sim f^2 BR^3\Omega^3 c^{-3} \sim 10^7 \text{Vcm}^{-1}$ . The radiation reaction limited energies of accelerator  $e^-/e^+$  are then large enough to give curvature radiation with  $\gamma$ -ray energies up to several GeV. Thus coming out of the starward end of the accelerator is a flow of  $\dot{N}_a \sim 10^{33} \text{s}^{-1}$  of  $e^-$  (or  $e^+$ ) which initially curvature radiate GeV  $\gamma$ -rays. Because  $\vec{E} \cdot \vec{B} \sim 0$  along the open field line path between the end of the accelerator and the polar cap, these  $e^-(e^+)$

lose much of their initial energy. They approach the polar cap with a Lorentz  $\gamma$  given by

$$\frac{1}{\gamma^3} - \frac{1}{\gamma_a^3} \sim \frac{2\Omega e^2}{mc^3} \ln \frac{r_a}{r}, \quad (2)$$

where  $\gamma_a$  is the initial  $\gamma$  when leaving the accelerator at a distance  $r_a \sim c\Omega^{-1}$  away from the star and  $r$  is the distance of the inflowing  $e^-(e^+)$  from the neutron star. The energy of the curvature radiated  $\gamma$ -rays when the star is approached by the inflowing particle beam is then

$$E_\gamma \sim \frac{\gamma^3 c^{1/2} \Omega^{1/2} \hbar}{r^{1/2}} \sim 10^2 \text{ MeV}. \quad (3)$$

The number of such curvature radiated  $\gamma$ -rays emitted over a distance  $(rc/\Omega)^{1/2}$  is

$$\frac{\dot{N}_\gamma}{\dot{N}_a} \sim \frac{\gamma e^2}{\hbar c} \sim 10^5 \quad (4)$$

for each inflowing  $e^-(e^+)$ .

As long as  $r \lesssim 10^7 \text{ cm}$  all of these  $\gamma$ -rays will be converted to  $e^\pm$  pairs by the neutron star magnetic field as they pass within a few stellar radii of the strongly magnetized star. However, from Eqs. (1) and (4) this would give

$$\dot{N}_\pm \sim 10^5 \dot{N}_a \sim 10^{38} \text{ s}^{-1}, \quad (5)$$

still two orders of magnitude less than needed to satisfy property 1 above. However, there can now be a second generation of  $e^\pm$  pairs which is produced from the synchrotron radiation which must accompany the  $\dot{N}_\pm$  production of Eq. (5). The  $\gamma$ -rays of Eqs. (3) and (4) are initially radiated with a  $\overrightarrow{\text{velocity}}$  almost exactly parallel to the  $\vec{B}$  along which their source  $e^-(e^+)$  moves. Because of  $\vec{B}$  field-line curvature the angle ( $\theta$ ) between the  $\gamma$ -ray momentum and the local  $\vec{B}$  increases as the  $\gamma$ -ray approaches the star. The  $\gamma$ -ray will be converted to an  $e^\pm$  pair as soon as

$$E_\gamma \sin \theta > 2mc^2, \quad (6)$$

if

$$B \gtrsim 10^{12} \text{ G}. \quad (7)$$

If Eq. (7) is not satisfied then Eq. (6) must be replaced by

$$E_\gamma \sin \theta \gtrsim 2mc^2/B_{12}. \quad (8)$$

From Eqs. (6) – (7) or Eq. (8) the energy of the synchrotron  $\gamma$ -rays from these  $e^\pm$  pairs is

$$\begin{aligned}\hbar\omega_s &\sim \left(\frac{E_\gamma}{mc^2}\right) \left(\frac{E_\gamma \sin \theta}{mc^2}\right) \left(\frac{\hbar e B}{mc}\right) \\ &\sim \left(\frac{E_\gamma}{mc^2}\right) (10 \text{ KeV}).\end{aligned}\tag{9}$$

From Eq. (3) we have  $\hbar\omega_s \sim 1 \text{ MeV}$  and a total pair production rate

$$\begin{aligned}\dot{N}_\pm(\text{total}) &= \dot{N}_\pm(\text{Eq.5}) \cdot \frac{E_\gamma(\text{Eq.3})}{\hbar\omega_s(\text{Eq.9})} \\ &\simeq \dot{N}_\pm(\text{Eq.5}) \cdot \left(\frac{mc^2}{10 \text{ KeV}}\right) \\ &\sim 10^{40} \text{ s}^{-1}.\end{aligned}\tag{10}$$

These pairs are produced all around the star within several stellar radii and we no longer have the problems of getting properties 1) – 3) from such an  $\dot{N}_\pm$  confined to the small open field line bundle above the polar cap.

We found that a large fraction of the  $e^\pm$  pairs produced in this way will annihilate (and at small velocities) at a distance about 17 km. from the center of the star. This is a consequence of the heating of the polar cap by the  $\dot{N}_a$  inflow of  $e^-$  ( $e^+$ ) down onto the polar cap. Each of these particles brings in about 6 ergs and therefore heats the polar cap area they impact upon to  $kT \sim 1 \text{ KeV}$ . Thus there is an emission from the polar caps of about  $6 \cdot 10^{33} \text{ erg s}^{-1}$  of X-rays with a typical energy  $E_x \sim 3 \text{ KeV}$ . This flux is too small to perturb greatly the  $e^\pm$  plasma flow except at the positions around the star where  $\hbar\omega_B \sim eB(r)\hbar/mc \cong 3 \text{ KeV}$ . At these  $r$  the X-ray-electron cross-section is resonant [ $\sigma \sim (2\pi^2 e^2/mc)\delta(\omega_B - \omega)$ ] and the resulting radiation pressure on the  $e^\pm$  plasma is very large. So is the inverse Compton drag there and  $e^-$  and  $e^+$  will tend to accumulate at special places where the net force of the radiation pressure is both perpendicular to the local  $\vec{B}$  and resonantly large. We found that most favored is an equatorial belt at

$$\left(\frac{R}{r}\right)^3 \sim \left(\frac{15 \text{ KeV}}{3 \text{ KeV}}\right) = 5\tag{11}$$

or

$$r \sim (1.7)R \sim 1.7 \times 10^6 \text{ cm},\tag{12}$$

when  $\hbar\omega_B$  on the stellar surface ( $r = R$ ) around the equatorial belt is about 15 KeV. This would give about the “observed” annihilation line redshift from the gravitational redshift at this distance  $r$  from a  $1.4M_\odot$  neutron star.

The annihilation line width is a critical issue, but it appears much more tractable than was the case in the previously considered models. In those cases  $\Delta E \sim (\Delta v/c)mc^2$  and it was difficult to see what process could bring to rest the initially rapid flowing ( $v \sim 10^{10} \text{cm s}^{-1}$ )  $e^\pm$  plasma to the needed precision. In the new model the resonant inverse compton drag from polar cap X-rays seems adequate. The main cause of  $\Delta E$  would then be the variation in  $GM/rc^2$  at resonance because  $|\vec{B}|$  depends upon angle as well as  $r$  and because of the spread in polar cap X-ray energies. In either case  $\Delta E \sim (\Delta r/r) \cdot 70 \text{ KeV}$  so that a considerable spread in dipolar  $B$  ( $\Delta B/B \sim 3/7$ ) in the “resonance layer” is tolerable. So far we have examined only the variation in  $GM/rc^2$  at fixed  $|B|$  because of angular variation in the resonance position  $r(\theta)$  and this does not give an unacceptable width.

Understanding the  $e^\pm$  environment around a  $\gamma$ -ray pulsar neutron star is a necessary preliminary for describing the soft X-rays which pass from the stellar surface through an outer-magnetosphere accelerator. It is especially important in deciding how that radiation is divided between direct polar cap emission ( $kT \sim 1 \text{ KeV}$ ) and general surface emission ( $kT \sim 10^{-1} \text{ KeV}$ ). The outer-magnetosphere accelerators for Vela, PSRs 1706 and 1055, and Geminga are proposed to be “bootstrapped” by  $X + \text{curvature } \gamma \rightarrow e^+ + e^-$ . These accelerator curvature  $\gamma$ -rays need be only energetic enough for  $E_x E_\gamma \gtrsim 2m^2 c^4$  and whether  $E_x \sim \text{few} \cdot 10^{-1} \text{ KeV}$  or  $\sim \text{few KeV}$  can be critical. The observed very high energy  $\gamma$ -rays from the above pulsars should be direct curvature  $\gamma$ -rays from  $e^-/e^+$  in the accelerator. A next step would be to use the observed soft X-ray emission to predict both the EGRET range  $\gamma$ -ray emission and the strength of possible  $e^\pm$  red-shifted annihilation lines in other  $\gamma$ -ray pulsars.

A first draft of a paper based upon the research described above is attached.

# Pair production in the magnetosphere of the Crab pulsar and a pulsed $e^\pm$ annihilation $\gamma$ -ray line

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## Abstract

Electron-positron pair production processes in polar cap and outer magnetosphere accelerator models are investigated for the Crab pulsar and compared to the properties of the reported  $e^\pm$  annihilation line from this pulsar. Polar cap models are found to be incapable of giving the  $10^{40}s^{-1}$   $e^\pm$  pair production rate needed for the gravitationally red-shifted  $e^\pm$  annihilation line in the spectrum reported by the Figaro collaboration. An  $e^\pm$  source related to an outer-magnetosphere accelerator model seems capable of giving this annihilation emission. A similar gravitational red-shifted  $e^\pm$  annihilation line may also be found to exist in the spectrum of the Geminga pulsar.

## 1 Introduction

Proposed pulsar magnetosphere models have different accelerator and pair production regions. In polar cap models, the accelerator forms relatively close to the neutron star surface within the open field line bundle. Charged particles are accelerated to extremely high energies (typically  $10^{12}$ eV) by the electric field of the accelerator. Moving along the curved magnetic field lines, these primaries emit typically  $10^2$ MeV curvature photons. Electron and positron pairs can be produced by those  $10^2$ MeV photons when they cross the strong neutron star magnetic field in the near magnetosphere:

$$\gamma + \mathbf{B} \rightarrow e^+ + e^- + \mathbf{B}. \quad (1)$$

The total potential drop of an polar cap accelerator is usually limited by a pair production cascade to about  $10^{12}$ Volts.

In outer-magnetosphere accelerator models, the charge depletion regions are located far away from neutron star surface and near the light cylinder (where a corotating magnetosphere would achieve a relativistic velocity). The local magnetic field is thus very much weaker than that in a polar cap model and  $e^\pm$  pair production mechanism of equation (1) is not relevant any more. Instead, the accelerator is controlled by pair production from collisions between  $\gamma$ -rays and other  $\gamma$ -rays or X-rays:

$$\gamma + \gamma \rightarrow e^+ + e^-. \quad (2)$$

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The total potential drop of an outer-magnetosphere accelerator is generally much larger than that of a polar cap accelerator.

In young pulsars such as the Crab, Vela and other  $\gamma$ -ray pulsars, both kinds of accelerators may exist. The  $e^\pm$  pair production rate and the annihilation location of the created  $e^\pm$  pairs from polar cap accelerator and outer-magnetosphere accelerator are very different. Thus a very strong gravitationally red-shifted  $e^+ + e^- \rightarrow \gamma + \gamma$  annihilation line in the spectrum of the Crab pulsar, reported by the Figaro collaboration, if confirmed, should discriminate between the two different models for the Crab pulsar. More generally, it would also help in understanding the current flow and pair production in a young pulsar's magnetosphere.

In section 2, we review briefly the observational history of a pulsed  $e^\pm$  annihilation line from the Crab pulsar, with special attention to the results from Figaro collaboration. Various problems in accounting for such annihilation feature in polar cap models are the main topics of section 3. In section 4, we consider this same gravitational red-shifted  $e^\pm$  annihilation line in outer-magnetosphere accelerator models. We find that the outer-magnetosphere accelerator model is favored by the observation (magnitude of redshift and strength of the line) of the Figaro collaboration.

## 2 Observational evidence for an $e^\pm$ annihilation line in the spectrum of the Crab pulsar

A possible  $\gamma$ -ray line at an energy close to, but less than, the rest energy of an electron had long been suspected since the discovery of the Crab pulsar. Such a red-shifted line would be expected from positron annihilation close to the surface of the Crab neutron star. The first observational report of such line came from Leventhal, MacCallum and Watts (1977) who claimed detection of an emission line at  $0.400 \pm 0.001$  MeV with an intensity of  $(2.2 \pm 0.9) \times 10^{-3}$  photons  $cm^{-2}s^{-1}$ . Since then various observations have either supported the existence of such line, but at a different intensity, or put a conflicting upper limit to its intensity. Some groups suggested variability in the annihilation rate.

A red-shifted  $e^+ + e^- \rightarrow \gamma + \gamma$  annihilation line in the spectrum of the Crab pulsar has been reported again by the Figaro collaboration. The Figaro experiment was especially designed to study pulsars and other sources with a well-established pulsed signature in the energy range 0.2-6.0 MeV. From both a July 1986 and a July 1990 observation, the Figaro collaboration reported a discovery (Agrinier B. et al and Massaro E. et al, 1990) of pulsed emission feature in the spectrum of Crab Pulsar (fig 1) with the following characteristics:

- The presumed  $e^\pm$  annihilation line is very strong. The observed flux of annihilation photons is  $F \simeq (0.86 \pm 0.33) \times 10^{-4}$  photons  $cm^{-2}s^{-1}$ , which implies an annihilation rate of  $10^{40} e^\pm s^{-1}$  in the magnetosphere of the Crab pulsar.
- The line is red-shifted. The annihilation line has an energy  $0.44 \pm 0.01$  MeV, red-shifted by 70 keV from the rest energy of an electron (or positron). This redshift from  $mc^2 \sim 0.51$  MeV has suggested that it escapes from the strong gravitational potential near the surface of the neutron star. If the Crab pulsar is assumed to have a mass  $1.4M_\odot$ , the annihilation radius which gives this redshift would be 16 km. On the other hand, if the annihilation happens at the surface of the neutron star with an assumed radius 10 km, the required mass is an implausibly small  $0.9M_\odot$ .
- The annihilation line is very narrow. The Figaro group's data is consistent with an emission line at energy 0.44 MeV with a half width of only 10 keV.



Thus any successful model for this reported gravitationally red-shifted annihilation line has to give an  $e^\pm$  pair production rate of  $10^{40} e^\pm s^{-1}$ . For an assumed  $1.4 M_\odot$  of the Crab pulsar mass, the annihilation region should be about  $1.6 \times 10^6 cm$  far from the center of the star. The maximum energy spread of the annihilation photons should not be bigger than 20 keV.

### 3 Difficulties in accounting for the reported $e^\pm$ annihilation line in polar cap models

Some kind of polar cap accelerator seems to be needed to give coherent radio emission from radio-pulsars. However, the existence of such accelerators don't exclude outer-magnetosphere accelerators on other bundles of "open field" lines. The following difficulties for polar cap models imply that the reported annihilation line would need an outer-magnetosphere accelerator in the Crab pulsar.

- A) The needed total  $e^\pm$  pair annihilation rate derived from the Figaro observational data is too big to accommodate a polar cap model explanation

The Goldreich-Julian charge density in the polar cap region of a pulsar is  $\rho \simeq \Omega B_p / 2\pi c$ . This gives an upper limit to the net particle flow through polar cap accelerator inside open field line bundle

$$I \leq \frac{\Omega \cdot B_p}{2\pi c e} \cdot c \cdot \pi R_p^2 = \frac{\Omega^2 B_p R^3}{2ce} \quad (3)$$

where  $\Omega$  is the spin rate of the pulsar,  $B_p$  is the magnetic field strength in the polar cap region and  $R_p = \sqrt{R^3 \Omega / c}$  is the maximum radius of the relevant open field bundle at the star surface. For the Crab pulsar this gives an upper limit of  $10^{34} s^{-1}$  for particle flow on pulsar's open field lines. Equation (3) is expected to hold for any surface field distribution with a dipole moment  $B_p R^3$  adjusted to fit to the pulsar spin-down rate. On the other hand, the total potential drop through the polar cap accelerator which is limited by  $e^\pm$  pair production cascade in the  $10^{12}$  gauss magnetic field to

$$\Delta V \leq 10^{12} Volts, \quad (4)$$

might be somewhat decreased if field line curvature radius is very much less than that for a pure central dipole. The total power extracted from a Crab pulsar polar cap accelerator  $I \Delta V \leq 10^{34} ergs^{-1}$ . Even with 100% efficiency for conversion of  $\gamma$ -rays into  $e^\pm$  rest mass, this could only marginally give the needed  $10^{40} e^\pm s^{-1}$  pair production rate. However, the actual conversion efficiency obtained for any plausible polar cap model is much smaller. Most of the curvature  $\gamma$ -ray energy would be transformed into  $e^\pm$  pairs by  $10^2$  MeV curvature  $\gamma$ -rays (Arons 1981, Sturrock 1971, Ruderman and Sutherland 1975), even if each  $10^2$  MeV  $\gamma$ -ray photon produces one pair the maximum  $e^\pm$  pair flux would be less than  $10^{34} erg s^{-1} / 10^2 MeV \sim 10^{38} s^{-1}$ . This is two orders of magnitude smaller than the pair flux needed for producing the reported annihilation line.

- B) The reported redshift of the annihilation line is not easily explained by the annihilation of  $e^\pm$  flux from a polar cap accelerator

It is plausible to attribute the redshift of the reported  $e^\pm$  annihilation line to the strong gravitational potential near the surface of the neutron star. However, to do so, the annihilating  $e^\pm$  pairs have to satisfy two conditions. (1) The electrons and positrons must annihilate well within two radii of the star. Otherwise, the gravitational redshift would be too small. (2) The  $e^\pm$  pairs must move at nonrelativistic speeds when they annihilate. If they move with relativistic speeds, the resulting Doppler shift of annihilation  $\gamma$ -rays would overwhelm the gravitational shift in almost all

directions and would not reproduce the narrowness of the 0.44 MeV feature in the Crab pulsed spectrum.

Electrons and positrons from polar cap accelerators satisfy neither of these conditions. They are almost all created moving out away from the surface of the star and would not annihilate appreciably before leaving the magnetosphere. If some positrons are turned back and flow into the star they could not give an inward particle flow greater than the  $10^{34} s^{-1}$  of equation (3) and this  $e^\pm$  annihilation rate is insignificant. Also most of these electron and positron pairs, produced by  $10^2$  MeV curvature  $\gamma$ -rays, move with extremely relativistic velocities: wherever they annihilate in flight the gravitational redshift would not be dominant.

- C) The positrons or electrons (depending on the angle between the spin axis and the magnetic moment) which are created in or pass through the accelerator itself may flow onto the polar cap and produce copious positrons through an  $e^\pm$  shower in the surface crust. However because of the high energy of these particles the resulting showers would develop so far below the polar cap surface that narrow  $e^\pm$  annihilation line emission from the surface would be negligible.

Thus a polar cap accelerator origin for the  $e^\pm$  pair production seems very unpromising for the reported  $e^\pm$  annihilation line. However, a rather delicate model (Bednarek, Cremonesi and Treves, 1992) which took use of the energy of the polar cap accelerator for  $e^\pm$  pair production has been proposed to account for the reported electron-positron annihilation line. In the model, a particle flow  $3 \times 10^{32} s^{-1}$  of primary electrons generated in the outer-magnetosphere accelerator was assumed to move toward the star along the magnetical field line of the open field bundle. Each of this electrons would produce about  $10^3$   $e^\pm$  pairs on its way toward the star. A significant fraction of the produced particles was assumed to enter into the polar cap accelerator, where each of them will get energy from the accelerator and radiate about  $10^5$  photons with energy close to 1 MeV. Once those photons get below the star surface they will be transformed into  $e^\pm$  pairs. The serious problem with this model is that they assumed a too big particle flow ( $3 \times 10^{32} \times 10^3 = 3 \times 10^{35}$ ) passing through the polar cap accelerator. Such a big charge particle flow will either quench the accelerator or cannot be sustained by the accelerator.

We consider next an outer magnetosphere accelerator and see if it can accommodate the Figaro group's observation.

## 4 $e^\pm$ pair production from an outer-magnetosphere accelerator with pair annihilation in the near-magnetosphere

The existence of an outer-magnetosphere accelerator in the Crab pulsar is supported strongly by several pieces of evidence. The pulsar's observed optical luminosity exceeds  $10^{-5}$  of its total spin-down power. The only incoherent radiation process which can give such high optical efficiency seems to be synchrotron radiation in a relatively weak magnetic field ( $B \leq 10^8 G$ ) which can be found only far ( $r \geq 4 \cdot 10^7 cm$ ) from the Crab stellar surface. However the precise phase coincidence of X-ray and  $\gamma$ -ray pulses with those of optical ones argues strongly for the same radiation source location, far from the star, i.e from an outer-magnetosphere accelerator. A model for a outer-magnetosphere accelerator of the Crab pulsar by Cheng, Ho and Ruderman (CHR 1986) indicates possible ways to fit the entire observed spectrum with such an accelerator.

If we attribute the large  $e^\pm$  pair production rate needed to account for the reported pulsed red-shifted  $e^\pm$  annihilation line of the Crab pulsar to an outer-magnetosphere accelerator, various problems of

polar cap models disappear. The total potential drop of outer-magnetosphere accelerator, no longer limited by  $\gamma + \mathbf{B} \rightarrow e^+ + e^- + \mathbf{B}$  pair production in the large  $\mathbf{B}$  near the star, can be much larger than that in a polar cap model. A very much greater fraction of a neutron star's spin-down power can then be extracted from an outer-magnetosphere accelerator. Many ways of producing  $e^\pm$  pairs remain. We consider now various  $e^\pm$  pair production processes in an outer-magnetosphere accelerator model in more detail. To illustrate our discussion, we use the geometry (fig.2) for a proposed Crab pulsar outer-magnetosphere accelerator (CHR 1986).

### A) Pair production inside the accelerator

Inside of the accelerator, electrons and positrons are accelerated to extremely high energy in opposite directions. The Crab pulsar accelerator field is expected to be balanced by curvature radiation reaction. For an electric field component  $\mathbf{E}$  along the accelerator, the limiting Lorentz factor  $\gamma$  of the primary electrons and positrons is then given by

$$e\mathbf{E}c \sim \frac{e^2}{c^3} \gamma^4 \left(\frac{c^2}{\rho}\right)^2, \quad (5)$$

where  $\rho$  is the curvature radius of the local magnetic field lines. In the Crab pulsar outer-magnetosphere accelerator equation (5) gives  $\gamma \sim 10^7$ . Curvature photons radiated by the primary electrons and positrons have an energy  $\sim \hbar\gamma^3(\frac{c}{\rho})$ . Essentially the full accelerator potential energy drop  $e\Delta V$  is radiated away as curvature radiation by the primary electrons and positrons. Each such electron and positron would produce about  $N_\gamma \sim \frac{\gamma e^2}{\hbar c} \sim 10^5$  curvature photons. To sustain the primary current flow and account for the observed X-ray and  $\gamma$ -ray luminosity, the needed primary particle flux is around  $10^{33} s^{-1}$ . Clearly annihilation of those  $e^\pm$  pairs makes a negligible contribution to the observed annihilation line.

### B) Pair production from GeV $\gamma$ -rays and surface X-rays

$e^\pm$  pairs can also be produced through collisions between very high energy  $\gamma$ -rays (greater than a GeV) from the outer-magnetosphere accelerator and soft thermal X-rays from the stellar surface of the Crab pulsar. However, due to the strong nonthermal synchrotron X-ray radiation from the secundaries, surface emission of soft X-rays from the Crab pulsar is probably relatively unimportant in producing  $e^\pm$  pairs. But, it may play a crucial role in other  $\gamma$ -ray pulsars such as the Vela, Geminga etc.

### C) Pair production from crossed outer-magnetosphere fan beams

Crossed fan photon beams would be formed from the almost symmetrical secondary  $e^\pm$  radiation as shown in Fig(2). Although in most of the KeV-GeV regime the crossed beams are optically thin to each other, photon-photon collisions in the crossed beams still produce abundant  $e^\pm$  pairs. For the Crab pulsar, a tertiary  $e^\pm$  population from crossed secondary photon beams also contributes to the high energy pulsed emission. Half of the pair flux (6 in Fig(2)) would flow out of the magnetosphere's light cylinder and the other half (7 in Fig(2)) would flow in along the open field line toward the star, which would be expected to annihilate close to the star. In the CHR outer-gap model for the Crab pulsar, the in-flow flux was estimated to be  $\dot{N}_\pm \sim 10^{39} e^\pm s^{-1}$ . With some stretch of the pulsar's outer-magnetosphere accelerator parameters, the model might accommodate an  $e^\pm$  pair production rate of  $\dot{N}_\pm \sim 10^{40} e^\pm s^{-1}$ . However the annihilation of this  $e^\pm$  source cannot satisfactorily explain the redshift and, especially, the narrow width of the reported annihilation line.

A  $10^{40} e^\pm s^{-1}$  pair production rate in the outer-magnetosphere region of the Crab pulsar would approach the stellar surface along the relatively small (area  $\leq 10^{11} cm^2$ ) bundle of open polar cap field lines. The cyclotron resonance interaction between in-flowing  $e^\pm$  plasma and surface X-rays might cause a shock

in the plasma close to the Crab stellar surface. If most electrons and positrons were to annihilate before passage through the shock, the speed of flow-in  $e^\pm$  would be

$$v \sim \sqrt{\frac{GM_{crab}}{r}} \sim 10^{10} \text{ cm s}^{-1}. \quad (6)$$

Such a large inflow speed would give annihilation line energy shifts varying greatly through the observed pulse (and thus an unacceptable huge line broadening). If the annihilation were to take place after passage through a shock standing above the surface, the post-shocked  $e^\pm$  plasma would be much too hot to give a narrow width of only 10 KeV. An  $e^\pm$  inflow of  $10^{40} \text{ s}^{-1}$  with the speed of equation (6), or less, would form an optically thick column ( $n_\pm \leq 10^{20} \text{ cm}^{-3}$  and  $n_\pm R_\rho \leq 10^{25} \text{ cm}^{-2}$ ) near the stellar surface. Redshifted annihilation  $\gamma$ -rays would then generally scatter before escape and the pulsed annihilation emission would be strongly diminished.

#### D) Inner-magnetosphere electron-positron pair production and annihilation from an outer-magnetosphere accelerator

The total X-ray and  $\gamma$ -ray emission from the Crab pulsar is about  $10^{-3}$  of that neutron star's spin-down power ( $-I\Omega\dot{\Omega}$ ). The scaling laws for the power from an outer-magnetosphere accelerator for the fraction of the Crab pulsar's open field line bundle spanned by its outer-magnetosphere accelerator then gives that fraction as  $f \sim 10^{-1}$ . The current flow through the accelerator ( $\dot{N}_a$ ) is then expected to be that same fraction of the total Goldreich-Julian current flow. For the Crab pulsar

$$\begin{aligned} \dot{N}_a &\sim f B_p R^3 \Omega^2 e^{-1} c^{-1} \\ &\sim f \cdot 10^{34} \text{ s}^{-1} \sim 10^{33} \text{ s}^{-1}. \end{aligned} \quad (7)$$

Within the accelerator this flow is accelerated by an electric field whose component along  $\mathbf{B}$  is of magnitude  $\sim f^2 B R^3 \Omega^3 c^{-3} \sim 10^7 \text{ V cm}^{-1}$ . The radiation reaction limited energies of accelerator  $e^-$  (or  $e^+$ ) are then large enough to give curvature radiation with  $\gamma$ -ray energies up to several GeV. Coming out of the starward end of the accelerator is a flow  $\dot{N}_a \sim 10^{33} \text{ s}^{-1}$  of  $e^-$  (or  $e^+$ ) which initially curvature radiate GeV  $\gamma$ -rays. Because  $\mathbf{E} \cdot \mathbf{B} \sim 0$  along the open field line path between the inward end of the accelerator and the polar cap, these  $e^-$  (or  $e^+$ ) lose most of their initial energy through curvature radiation. The instantaneous energy loss rate is

$$\dot{E} = \dot{\gamma} m c^2 = -\frac{2c}{3} \left(\frac{e}{\rho}\right)^2 \gamma^4 \quad (8)$$

where  $\rho$  is the local curvature radius of the field lines which link the accelerator to the polar cap. For a dipole field  $\rho \sim (rc/\Omega)^{1/2}$ . When the primaries approach the polar cap, their Lorentz  $\gamma$  is given by

$$\frac{1}{\gamma^3} - \frac{1}{\gamma_a^3} \sim \frac{2\Omega e^2}{mc^3} \ln \frac{r_a}{r} \quad (9)$$

where  $\gamma_a$  is  $\gamma$  when leaving the accelerator at a distance  $r_a \sim c\Omega^{-1}$  away from the star and  $r$  is the distance of the  $e^-$  ( $e^+$ ) from the neutron star. As long as  $r \ll r_a$ ,  $\gamma$  is not sensitive to the distance  $r$  and is approximately constant along the electron trajectory. For the Crab pulsar near the star

$$\gamma \sim \left( \frac{2\Omega e^2}{mc^3} \ln \frac{r_a}{r} \right)^{1/3} \sim 5 \times 10^6. \quad (10)$$

The energy of each curvature radiated  $\gamma$ -ray as the star is approached,

$$E_\gamma \sim \frac{\gamma^3 c^{1/2} \Omega^{1/2} \hbar}{r^{1/2}} \sim 60 \left( \frac{r}{10^7} \right)^{1/2} \text{ MeV} \quad (11)$$

The number of such  $\gamma$ -rays radiated over a distance  $(rc/\Omega)^{1/2}$  is

$$\frac{\dot{N}_\gamma}{\dot{N}_a} \sim \frac{\gamma e^2}{\hbar c} \quad (12)$$

for each inflowing  $e^- (e^+)$ . These  $\gamma$ -rays are radiated in a direction almost exactly parallel to the  $\mathbf{B}$  along which their source  $e^- (e^+)$  moves. Because of the curvature of the magnetic field line the angle  $\theta$  between a  $\gamma$ -ray's momentum and the local magnetic field through which it passes increases as the  $\gamma$ -ray approaches the star. The  $\gamma$ -ray will be converted to an  $e^\pm$  pair as soon as

$$\frac{E_\gamma \sin \theta}{2mc^2} \geq 1, \quad (13)$$

if

$$B \geq 2 \times 10^{12} \text{ Gauss}. \quad (14)$$

If equation (14) is not satisfied then equation (13) must be replaced by

$$\frac{E_\gamma \sin \theta}{2mc^2} \cdot \frac{B}{2 \times 10^{12}} \geq 1. \quad (15)$$

The energy of each of the materialized electrons and positrons is about half of the parent  $\gamma$ -ray energy. The electrons and positrons would lose almost all their momentum perpendicular to  $\mathbf{B}$  immediately, through synchrotron radiation in the strong magnetical field. From equations (13)-(14) or equation (15) the typical energy of the synchrotron  $\gamma$ -rays

$$\begin{aligned} \hbar\omega_s &\sim \left(\frac{E_\gamma}{2mc^2}\right) \left(\frac{E_\gamma \sin \theta}{2mc^2}\right) \left(\frac{\hbar e B}{mc}\right) \\ &\sim \left(\frac{E_\gamma}{2mc^2}\right) (20 \text{ KeV}). \end{aligned} \quad (16)$$

These synchrotron  $\gamma$ -rays would themselves be converted into a second generation of  $e^\pm$  pairs if the following two criteria are satisfied.

1. The energy of the synchrotron photons exceeds the threshold energy,  $2m_e c^2 \sim 1 \text{ MeV}$ , for pair production. From equation (16) this leads to

$$E_\gamma \geq 50 \text{ MeV}. \quad (17)$$

2. The synchrotron  $\gamma$ -rays pass accross a strong magnetic field, of order  $10^{12} \text{ Gauss}$ . We can assume that this condition is satisfied if the synchrotron photon trajectory passes within two stellar radii of the neutron star. For a pure dipole magnetic field this condition is met if the radial distance from the star at which the primary curvature  $\gamma$ -ray was emitted satisfies

$$\frac{1}{2} \sqrt{\frac{r^3 \Omega}{c}} \leq 2R \quad (18)$$

or

$$r \leq (16R^2 \frac{c}{\Omega})^{1/3} \sim 1.4 \times 10^7. \quad (19)$$

After inserting equation (19) into equation (11), we find that  $E_\gamma \geq 50 \text{ MeV}$  which is just the inequality of equation (17). Thus when  $r \leq 1.4 \times 10^7 \text{ cm}$ , both of conditions (1) and (2) are satisfied and almost all of the synchrotron  $\gamma$ -rays from the first  $e^\pm$  pair generation are themselves converted into  $e^\pm$  pairs. From equations (7),(10),(12),(16) and (19), the total electron-positron production rate

$$\begin{aligned}\dot{N}_\pm(\text{total}) &= 2\dot{N}_a \cdot \frac{\gamma e^2}{\hbar c} \cdot \frac{E_\gamma}{\hbar \omega_s} \\ &\simeq 5 \times 10^{39} \text{ s}^{-1}.\end{aligned}\tag{20}$$

This is the electron-positron production rate from only one pole; the total production rate from both poles is about  $10^{40} \text{ s}^{-1}$  in agreement with the Figaro observation. These electrons and positrons are produced all around the star within several radii of it and we no longer have the problems associated with such an  $\dot{N}_\pm$  confined to the very small open field line bundle above the polar cap, the result from crossed fan beams. It is this widely spread  $e^\pm$  plasma that we propose for the source of the reported  $e^\pm$  annihilation line. The corresponding annihilation region and the width of the resulting annihilation line are discussed below.

### E) The gravitational energy shift and width of the annihilation line

The polar cap area is impacted by the primary inflow ( $\dot{N}_a$ ) of  $e^-$  ( $e^+$ ). Each of these particles brings in

$$E(R) \simeq mc^2 \left[ \frac{2\Omega e^2}{mc^3} \ln\left(\frac{r_a}{r}\right) \right]^{-1/3} \sim 4 \text{ ergs} \tag{21}$$

and heats the polar cap area they impact to  $\text{kT} \sim 1 \text{ keV}$ . There is then an expected emission from the Crab pulsar's polar caps of about  $4 \cdot 10^{33} \text{ ergs}^{-1}$  of soft X-rays of a typical energy  $E_x \sim 3 \text{ keV}$ . This flux is too small to perturb greatly the  $e^\pm$  plasma flow except at these positions around the star where

$$\hbar \omega_B \equiv \hbar \frac{eB(r)}{mc} \simeq 3 \text{ KeV}.\tag{22}$$

For those places where  $r$  satisfying equation (22), the scattering cross-section of X-rays and electrons is resonant [ $\sigma \sim (2\pi^2 e^2/mc)\delta(\omega_B - \omega)$ ] and the radiation pressure on the  $e^\pm$  plasma can be very large. So is the inverse Compton drag. Then  $e^-$  and  $e^+$  would tend to accumulate at those special places in the near-magnetosphere where the net force of the radiation pressure is perpendicular to the local  $\mathbf{B}$  and also resonantly large. For simplicity, we assume a dipole magnetic field outside of the star:

$$\begin{aligned}B(r, \theta) &= \frac{B_p}{2} \left(\frac{R}{r}\right)^3 [2 \cos \theta \vec{e}_r + \sin \theta \vec{e}_\theta] \\ &\simeq 1.5 \times 10^{12} \left(\frac{R}{r}\right)^3 [2 \cos \theta \vec{e}_r + \sin \theta \vec{e}_\theta]\end{aligned}\tag{23}$$

where  $B_p \sim 3 \times 10^{12} \text{ gauss}$ , the magnetic field at the poles, fits the observed stellar spin-down power. From equations (22) and (23) the resonant surface  $r(\theta)$  is given by

$$\begin{aligned}\left(\frac{R}{r(\theta)}\right)^3 &= \frac{\hbar \omega \cdot mc}{e\hbar \cdot B_p (\cos^2 \theta + \frac{1}{4} \sin^2 \theta)^{1/2}} \\ &\simeq \frac{1}{5(4 \cos^2 \theta + \sin^2 \theta)^{1/2}}.\end{aligned}\tag{24}$$

or

$$r(\theta) \simeq 1.7R[1 + 3 \cos^2 \theta]^{1/6}\tag{25}$$

The most favored annihilation region is then found to be an equatorial belt at

$$r \sim 1.7R \sim 1.7 \times 10^6 \text{ cm}. \quad (26)$$

This gives about the observed annihilation line redshift as the gravitational redshift at this distance  $r$  from a  $1.4M_\odot$  neutron star.

Because most of the  $e^-$  and  $e^+$  are brought to very nonrelativistic speeds by large inverse Compton drag, the main cause of the annihilation line width could then be the variation in  $GM/rc^2$  at resonance because  $|\mathbf{B}|$  depends upon angle  $\theta$  as well as  $r$ , and because of the spread in polar cap X-ray energies. For a monoenergetic polar cap X-ray emission at 3keV, the maximum energy spread of the annihilation photons is, from equation (25),

$$\Delta E \sim (\Delta r/r) \cdot 70 \text{ KeV} \sim 0.26 \times 70 \text{ KeV} \sim 20 \text{ KeV}, \quad (27)$$

corresponding to a half-width of about 10keV. This will, of course, be increased because of energy spread in polar cap X-rays. However, it seems possible to preserve a narrow annihilation line.

## 5 Summary and discussion

We have compared the different  $e^\pm$  pair production processes from polar cap models and outer-magnetosphere accelerator models and found that while it seems very difficult for polar cap models to account for the narrow gravitational red-shifted annihilation line reported by the Figaro collaboration, an outer-magnetosphere accelerator model can reproduce crucial aspects of the observational data.

The Geminga pulsar may also have an observable gravitational red-shifted  $e^\pm$  annihilation line in its spectrum because of a powerful accelerator in its outer-magnetosphere. Due to this pulsar's high efficiency for radiating  $\gamma$ -ray photons, however, (unlike the case of the Crab pulsar) Geminga's outer-magnetosphere accelerator would span almost all of accessible open field lines. The primary particle flow through the accelerator is then almost equal to its Goldreich-Julian current flow

$$\dot{N}_a \sim \frac{B_p R^3 \Omega^2}{ec} \sim 8 \times 10^{31} \text{ s}^{-1}. \quad (28)$$

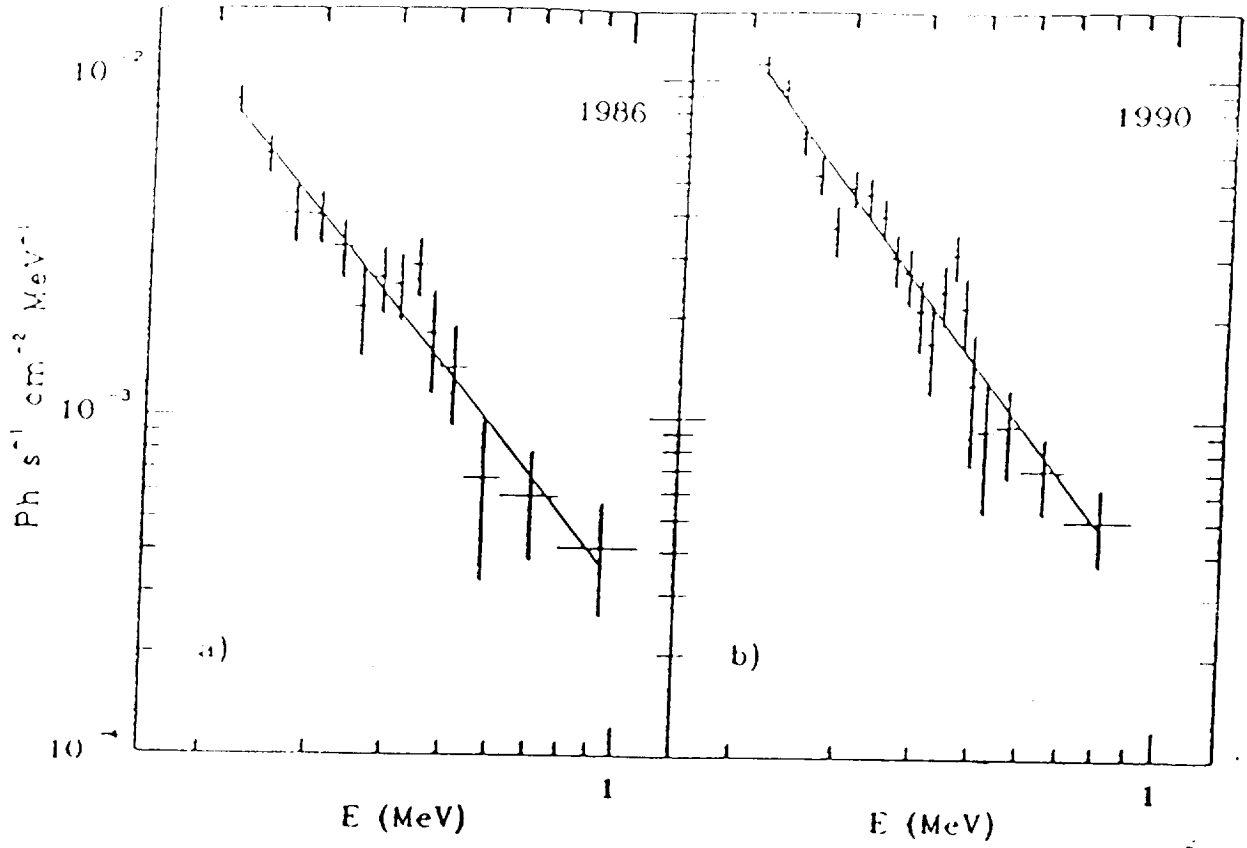
With a model similar to that for the Crab pulsar, the  $e^\pm$  pair production rate in the near magnetosphere region of Geminga pulsar by the the flow-in primary  $e^-$  ( $e^+$ ) could be as high as  $8 \times 10^{38} \text{ s}^{-1}$ , about one order of smaller than that of the Crab pulsar. However at an estimated distance only about 250pc (Halpern et al 1993), the Geminga pulsar is much closer. The possibly observable  $e^\pm$  annihilation photon flux from the Geminga pulsar might give an observed line even slightly stronger than that of the Crab pulsar. A crucial difference, however, may lie in the relative strengths of their outer-magnetosphere magnetic fields. Only when it is very strong, as in the case of the Crab pulsar, will the standard flow of  $e^-$  ( $e^+$ ) from the accelerator be initially only along the local  $\mathbf{B}$ . If tilted angle between the rotation axis and the magnetic moment of the Gemingar pulsar is not very big, the outer-magnetical acclerator would locate rather far from the star, where the local magnetical field could be quite low compared with that of the Crab pulsar. Then the momentum of the primary  $e^-$  ( $e^+$ ) could have a significant component (how mach depends on how pairs are made within the accletator) perpendicular to direction of the local magnetical field. This perpendicular compoment would be radiated away rather quickly when the primary  $e^-$  ( $e^+$ ) move out of the accelerator toward the star and get into a stronger magnetical field region. The subsequent energetic  $\gamma$  ray emission near the star

could then be greatly reduced and an  $e^\pm$  annihilation line would unlikely be expected from this pulsar. This may also be why the observed total X-ray luminosity from the Gemingar pulsar is less than the expected heating (Hapler & Ruderman,1993) from the impacting primary electrons (positrons) when these primary particles are assumed to have no perpendicular momentum with respect to the local magnetic field.

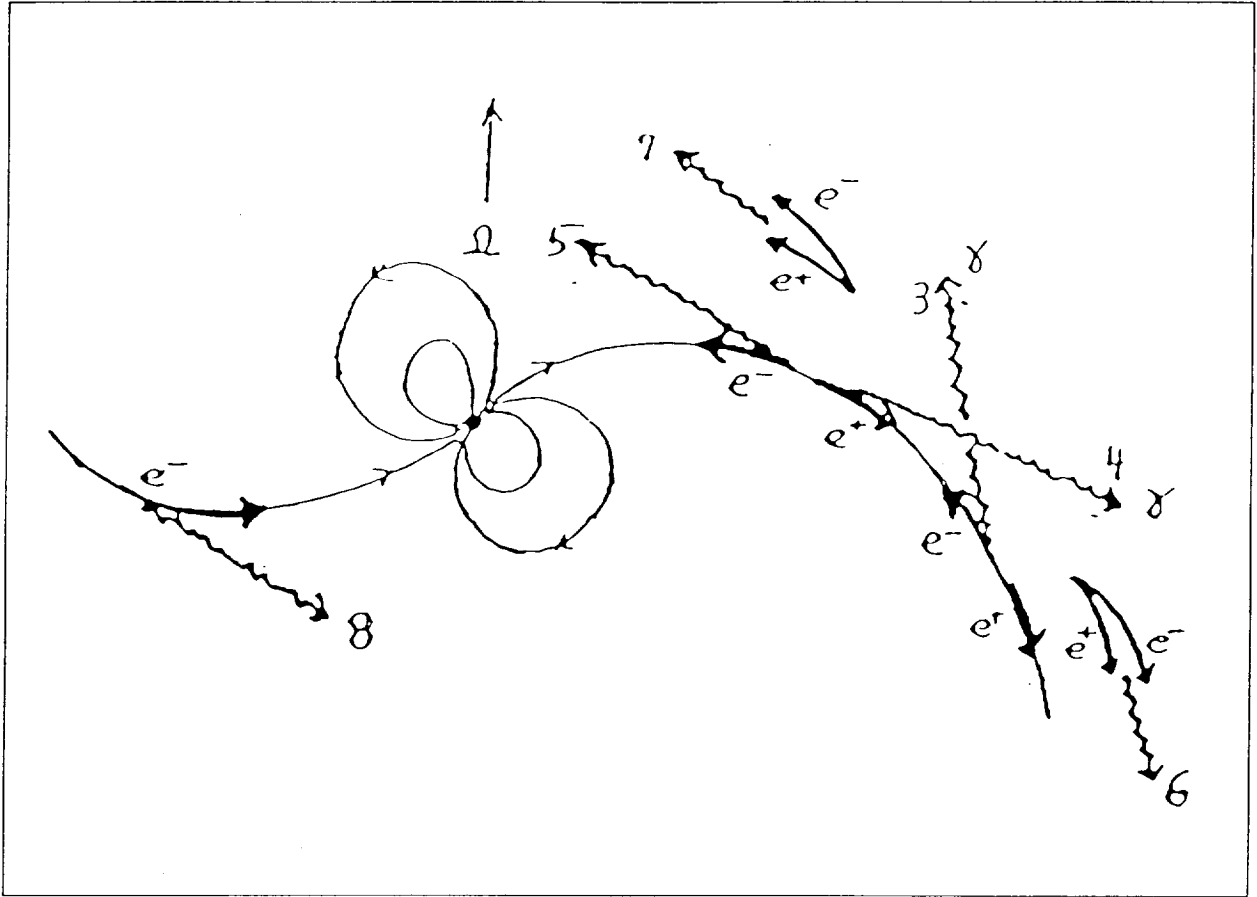
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**Fig. 1.** The net spectra of the Crab photons after subtraction of the off-pulse signal, for the two FIGARO II flights of (a) 1986 July 11 and (b) 1990 July 9. From E. Massaro et al., *Ap.J.*, 376, L11 (1991).



**Fig. 2.** The geometry for photon emission from outer-magnetosphere accelerators which accelerate  $e^-/e^+$  along local  $\mathbf{B}$  with an electric field component  $\mathbf{B} \times \mathbf{E}$  within the accelerator. The resulting radiation from curvature radiation, synchrotron radiation, or inverse Compton scattering is a fan covering almost all latitudes. Similar beams go outward (beams 4,6) and inward (beams 3,7). Because of dipolar symmetry the observer who sees outward moving photons in beam 4 in one subpulse will also see inward moving photons (beam 8) from the other side of the star in a later arriving second subpulse as the star rotates. Gamma-ray beam crossing such as shown for beams 3 and 4 will sustain  $e^\pm$  production around the accelerator.